Detecting Adaptive Differentiation in Structured Populations with Genomic Data and Common Gardens

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ABSTRACT Adaptation in quantitative traits often occurs through subtle shifts in allele frequencies at many loci—a process called polygenic adaptation. While a number of methods have been developed to detect polygenic adaptation in human populations, we lack clear strategies for doing so in many other systems. In particular, there is an opportunity to develop new methods that leverage datasets with genomic data and common garden trait measurements to systematically detect the quantitative traits important for adaptation. Here, we develop methods that do just this, using principal components of the relatedness matrix to detect excess divergence consistent with polygenic adaptation, and using a conditional test to control for confounding effects due to population structure. We apply these methods to inbred maize lines from the United States Department of Agriculture germplasm pool and maize landraces from Europe. Ultimately, these methods can be applied to additional domesticated and wild species to give us a broader picture of the specific traits that contribute to adaptation and the overall importance of polygenic adaptation in shaping quantitative trait variation.

KEYWORDS Local adaptation; polygenic adaptation; quantitative genetics; population genetics; maize

DETERMINING the traits involved in adaptation is crucial for understanding the maintenance of variation (Mitchell-Olds *et al.* 2007), the potential for organisms to adapt to climate change (Aitken *et al.* 2008; Bay *et al.* 2017), and the best strategies for breeding crops or livestock (Howden *et al.* 2007; Takeda and Matsuoka 2008). There are many examples of local adaptation from reciprocal transplant experiments (Leimu and Fischer 2008; Hereford 2009) that tell us about fitness in a specific environmental context, but these experiments are less informative about how past evolutionary forces have shaped present day variation (Savolainen *et al.* 2013). Instead, quantifying the role of adaptation in shaping current phenotypic variation will require comparing observed variation with expectations based on neutral models (Leinonen *et al.* 2008). With the growing number of large genomic and phenotypic common garden

datasets, there is an opportunity to use these types of comparisons to systematically identify the traits that have diverged due to adaptation.

A common way of evaluating the role of spatially variable selection in shaping genetic variation is to compare the proportion of the total quantitative trait variation among populations (Q_{ST}) with that seen at neutral polymorphisms (F_{ST}) (Prout and Barker 1993; Spitze 1993; Whitlock 2008). $Q_{ST} - F_{ST}$ methods have been successful at identifying local adaptation, but have a few key limitations that are especially important for applications to large genomic and phenotypic datasets (Whitlock 2008; Leinonen *et al.* 2013). First, standard $Q_{ST} - F_{ST}$ assumes a model in which all populations are equally related [but see Whitlock and Gilbert (2012), Ovaskainen *et al.* (2011), Karhunen *et al.* (2013) for methods that incorporate more complicated models of population structure]. Second, rigorously estimating Q_{ST} requires knowledge of the additive genetic variance V_A both within and between populations (Whitlock 2008). Many studies skirt this demand by measuring the proportion of phenotypic variation partitioned between populations ("P_{ST}"), either in natural habitats or in common gardens. However, replacing Q_{ST} with P_{ST} can lead to problems due to both environmental differences

among natural populations and nonadditive variation in common gardens (Pujol *et al.* 2008; Whitlock 2008; Brommer 2011). Third, $Q_{ST} - F_{ST}$ approaches are unable to evaluate selection in individuals or populations that have been genotyped but not phenotyped. It can be cheaper to phenotype in a smaller panel and test for selection in a larger genotyped panel. If individuals are heterozygous or outbred, cannot be easily maintained in controlled conditions, or are dead, they can be genotyped but not easily phenotyped. In these cases, the population genetic signature of adaptation in quantitative traits ("polygenic adaptation") can be detected by looking for coordinated shifts in the allele frequencies at loci that affect the trait (Latta 1998; Le Corre and Kremer 2012; Kremer and Le Corre 2012).

Current approaches to detect polygenic adaptation in genomic data take advantage of patterns of variation at large numbers of loci identified in genome-wide association studies (GWAS) (Turchin *et al.* 2012; Berg and Coop 2014; Field *et al.* 2016). One approach, Q_X, developed by Berg and Coop (2014), extends the intuition underlying classic $Q_{ST} - F_{ST}$ approaches by generating population-level polygenic scores—trait predictions generated from GWAS results and genomic data—and comparing these scores to a neutral expectation. However, methods for detecting polygenic adaptation using GWAS-identified loci are very sensitive to population structure in the GWAS panel (Berg and Coop 2014; Robinson *et al.* 2015; Berg *et al.* 2018; Novembre and Barton 2018; Sohail *et al.* 2018). Because GWAS in many systems are conducted in structured, species-wide panels (Flint-Garcia *et al.* 2005; Atwell *et al.* 2010; Wang *et al.* 2018), current methods for detecting polygenic adaptation are difficult to apply widely.

Here, we adapt methods for detecting polygenic adaptation to be used in structured GWAS panels and related populations. First, using a new strategy for estimating V_A, we develop an extension of $Q_{ST} - F_{ST}$, which we call Q_{PC}, to test for evidence of adaptation in a heterogeneous, range-wide sample of individuals that have been genotyped and phenotyped in a common garden. We then develop an extension of Q_X for use in structured GWAS populations where the panel used to test for selection shares population structure with the GWAS panel. We apply both of these methods to data from domesticated maize (*Zea mays* ssp. *mays*). Overall, we show that the method controls for false positive issues due to population structure and can detect selection on a number of traits in domesticated maize.

Results

Extending $Q_{ST} - F_{ST}$ to deal with complicated patterns of relatedness with Q_{PC}

Our approach to detecting local adaptation is meant to ameliorate two main aspects of $Q_{ST} - F_{ST}$ analysis that limit its application to many datasets. First, many species-wide genomic datasets are collected from individuals that do not group

naturally into populations, making it difficult to look for signatures of divergence between populations. Second, calculating Q_{ST} requires an estimate of V_A, usually done by phenotyping individuals from a crossing design.

We address these issues by using principal component analysis (PCA) to separate the kinship matrix, K, into a set of principal components (PCs) that are used to estimate V_A, and an orthogonal set of PCs used to test for selection. We base our use of PCA on the animal model, which is often used to partition phenotypic variance among close relatives within populations into genetic and environmental components (Henderson 1950, 1953; Thompson 2008). More generally, the animal model is a statement about the distribution of an additive phenotype if the loci contributing to the trait are drifting neutrally (see Ovaskainen *et al.* 2011; Berg and Coop 2014; and Hadfield and Nakagawa 2010).

We first use the animal model to describe how traits are expected to vary across individuals under drift alone. Let $\overrightarrow{Z}$ be a vector of measurements for a given trait across M individuals, taken in a common garden with shared environment. Assume for the moment that this trait is made up only of additive genetic effects, that environmental variation does not contribute to trait variation ($V_P = V_A$), and that measurements are without error (*i.e.*, that $\overrightarrow{Z}$ are breeding values). The animal model then states that $\overrightarrow{Z}$ has a multivariate normal distribution:

$$\overrightarrow{Z} \sim MVN(\mu, 2V_A K), \tag{1}$$

where μ is the mean phenotype, V_A is the additive genetic variance, and K is a centered and standardized $M \times M$ kinship matrix, where diagonal entries represent the inbreeding coefficients of individuals and off-diagonal cells represent the genotypic correlations between individuals (see Equation 16 in *Materials and Methods*). The kinship matrix describes how variation in a neutral additive genetic trait is structured among individuals due to variation in relatedness, while V_A describes the scale of that variation.

Before discussing how we can use Equation 1 to develop a test for adaptive divergence, we will show how this equation relates to $Q_{ST} - F_{ST}$. If the individuals in our sample are grouped into a set of P distinct populations, the kinship matrix can be used to generate an expectation of how trait variation is structured among populations under neutrality. The vector of population mean breeding values ($\overrightarrow{Z}_{\text{pop}}$) can be calculated from individual breeding values as $\overrightarrow{Z}_{\text{pop}} = H^T \overrightarrow{Z}$, where the pth column of the $M \times P$ matrix H has entries of $\frac{1}{n_p}$ for individuals sampled from population p, and 0 otherwise (n_p is the number of individuals sampled from population p). Because $\overrightarrow{Z}$ is multivariate normal, it follows that $\overrightarrow{Z}_{\text{pop}}$ is as well, with

$$\overrightarrow{Z}_{\text{pop}} \sim MVN\left(\mu_{\text{pop}}, 2V_A K_{\text{pop}}\right), \tag{2}$$

where $K_{\text{pop}} = H^T K H$ and μ_{pop} is the mean trait value across all populations.

Based on Equation 2, we can calculate a simple summary statistic describing the deviation of $\vec{Z}_{\text{pop}}$ from the neutral expectation based on drift:

$$Q_X = \frac{\left(\vec{Z}_{\text{pop}} - \mu\right)^T K_{\text{pop}}^{-1} \left(\vec{Z}_{\text{pop}} - \mu\right)}{2V_A}. \tag{3}$$

Under neutrality, Q_X is expected to follow a χ^2 distribution with $P - 1$ degrees of freedom (μ is not known *a priori* and must be estimated from the data, which expends a degree of freedom) (Berg and Coop 2014). If all P populations are diverged equally from one another, with no additional structure or inbreeding within groups, then $K_{\text{pop}} = F_{ST}I$, where F_{ST} is a measure of genetic differentiation between the populations and I is the identity matrix. Then, Equation 3 simplifies to

$$\begin{aligned} Q_X &= \frac{(P-1)\text{Var}\left(\vec{Z}\right)}{2V_A F_{ST}} \\ &= (P-1)\frac{Q_{ST}}{F_{ST}} \end{aligned} \tag{4}$$

showing that the Q_X statistic described in Equation 3 is the natural generalization of $Q_{ST} - F_{ST}$ to arbitrary population structure.

Here, we test for selection by looking for excess phenotypic divergence along the major axes of relatedness described by PCs instead of looking for excess divergence between populations. For an example of how PCs relate to within and between population variation in a set of simulated populations, see Figure 1. In these simulated populations, PCs 1 and 2 distinguish between-population variation for three populations, while PC 3 separates out individuals within one population. While this is a simplified example compared to the populations analyzed later in the paper, it shows the intuition underlying our use of PCs to replace the within and between population structure used by $Q_{ST} - F_{ST}$.

Q_X can be linked to a PC-based approach by noting that, for any arbitrary H matrix (not just the type described above), Q_X will follow a χ^2 distribution, and the degrees of freedom of this distribution will be equal to the number of linearly independent columns in H. We can generate a measure of excess divergence along PCs by replacing H with a matrix, U, of eigenvalues of the kinship matrix K. U is calculated from the eigendecomposition of K, such that $K = U\Lambda U^T$, where $U = [\vec{U}_1, \vec{U}_2, \vec{U}_3, \ldots \vec{U}_M]$ is the matrix of eigenvectors and Λ is a diagonal matrix with the eigenvalues of K. Here, we denote the mth eigenvector as $\vec{U}_m$ and the mth eigenvalue as λ_m, and we call our measure of excess divergence along PCs Q_{PC}.

We will now walk through calculating Q_{PC}. First we quantify the amount of divergence that occurs along PCs by projecting the trait described by $\vec{Z}$ onto the eigenvectors of K by letting $z_m = (\vec{Z} - \mu) \cdot \vec{U}_m$. Intuitively, z_m describes how much the traits ($\vec{Z}$) vary along the mth PC of the relatedness matrix K. z_m can also be thought of as the slope of the re-

lationship between $\vec{Z}$ and the mth PC of K. Under a neutral model of drift (from Equation 1) for each m:

$$z_m \sim N(0, 2V_A\lambda_m). \tag{5}$$

To compare z_m across different PCs, we can calculate a standardized projection (c_m):

$$c_m = \frac{z_m}{\sqrt{\lambda_m}}, c_m \sim N(0, 2V_A). \tag{6}$$

Crucially, c_m values represent deviations along linearly independent axes of neutral variation (PCs) and are independent from each other under neutrality; therefore, we can estimate V_A using the variance of any set of c_m. To develop a test analogous to $Q_{ST} - F_{ST}$, we choose to declare projections onto the top $1:R$ of our eigenvectors ($\vec{C}_{1:R}$) that explain broader patterns of relatedness to be "among population" axes of variation, and projections onto the lower $R + 1 : M$ of our eigenvectors ($\vec{C}_{R+1:M}$) to be "within population" axes of variation (we will discuss our choice of R later).

Under neutrality, we expect that $\text{Var}(\vec{C}_{1:R}) = \text{Var}(\vec{C}_{R+1:M})$. Adaptive differentiation among populations will increase the trait variance explained by the first PCs relative to the variance explained by later PCs and $\text{Var}(\vec{C}_{1:R}) > \text{Var}(\vec{C}_{R+1:M})$. Note that $\text{Var}(\vec{C})$ is the same as $\mathbb{E}[\vec{C}^2]$ since the mean of $\vec{C}$ is 0 based on Equation 6. We can test for differences between the variance of projections onto early PCs and the variance of projections later PCs using an F test:

$$Q_{\text{among:within}} = \frac{\mathbb{E}[\vec{C}_{1:R}^2]}{\mathbb{E}[\vec{C}_{R+1:M}^2]} \sim F(R, M - R). \tag{7}$$

We focus on the upper tail of the F distribution, as we are interested in testing for evidence of selection contributing to trait divergence. Rejection of the null indicates excess trait variation in the first R PCs beyond an expectation based on the later $M - R$ PCs. All together, this test allows us to detect adaptive trait divergence across a set of lines or individuals without having to group these individuals into specific populations.

We can also calculate variance along specific PCs and compare divergence along specific PCs to the additive variance estimated using the lower $R : M$ eigenvectors. Looking at specific PCs will be useful for identifying the specific axes of relatedness variation that drive adaptive divergence as well as for visualizing results. For a given PC, S:

$$Q_{PC} = \frac{\mathbb{E}[C_S^2]}{\mathbb{E}[\vec{C}_{R+1:M}^2]} \sim F(1, M - R). \tag{8}$$

The rejection of the null corresponds to excess variance along the sth PC beyond expectations based on variance along lower PCs. Equation 7 and Equation 8 are valid for any values of S, R, and M as long as $R > S$ and $M > R$. However, picking values of S, R, and M may not be trivial. We test for excess
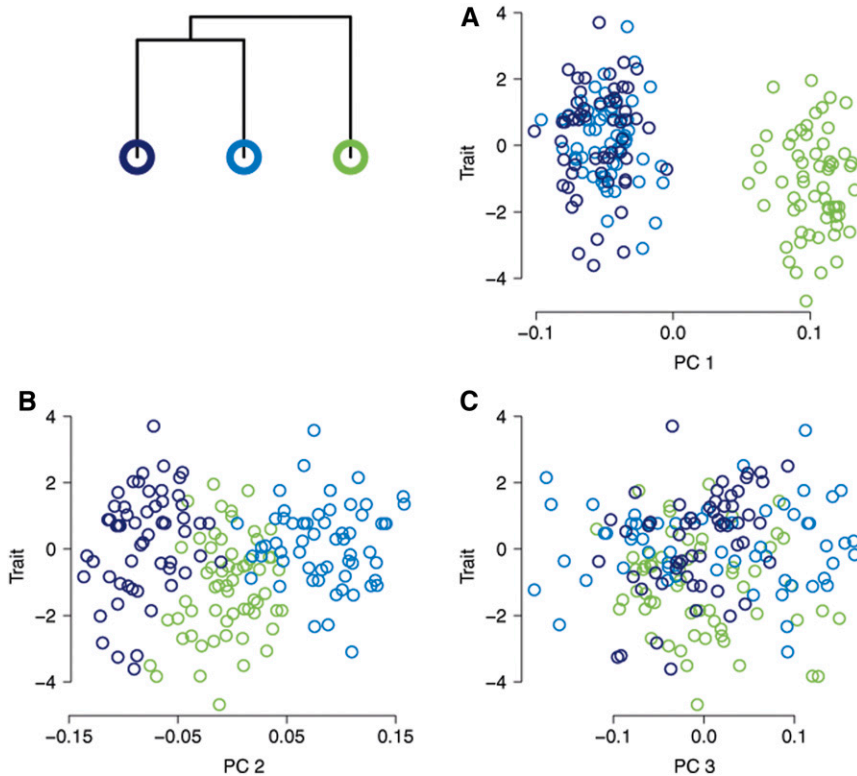
Figure 1 Relating PCs to within- and between-population variation. A conceptual figure based on three populations simulated following the dendrogram on the left of plot A. Each population has 70 individuals in it with some within-population variation in relatedness (data not shown.). (A–C) The relationship between PCs 1, 2, and 3 and a neutrally evolving trait.

differentiation along the first set of PCs that cumulatively explain 30% of the total variation in relatedness. However, an alternative that we do not explore here would be selecting the set of PCs to use with methods from Bryc *et al.* (2013) or the Tracy-Widom distribution discussed in Patterson *et al.* (2006).

Testing for selection with Q_PC in a maize mapping panel

We applied Q_{PC} to test for selection in a panel of 240 inbred maize lines from the GWAS panel developed by Flint-Garcia *et al.* (2005). The GWAS panel includes inbred lines meant to represent the diversity of temperate and tropical lines used in public maize breeding programs, and these lines were recently sequenced as part of the maize HapMap 3 project (Bukowski *et al.* 2017). In Figure 2A, we plot the relatedness of all maize lines on the first two PCs. The first PC explains 2.04% of the variance and separates out the tropical from the nontropical lines, while the second PC explains 1.90% of the variance and differentiates the stiff-stalk samples from the rest of the dataset [stiff-stalk maize is one of the major heterotic groups used to make hybrids (Mikel and Dudley 2006)]. While previous studies have used relatedness to assign lines to subpopulations, not all individuals can be easily assigned to a subpopulation, and there is a fair amount of variation in relatedness within subpopulations (Flint-Garcia *et al.* 2005) (Figure 2A).

We first validated that Q_{PC} would work on this panel by testing Q_{PC} on 200 traits that we simulated under a multivariate normal model of drift based on the empirical kinship matrix, assuming $V_A = 1$. As expected, from Equation 6, the variance in the standardized projections onto PCs (c_m) of these simulated traits centered on 1, and, across the 22 PCs tested in 200 simulations, only 200 tests (4.5%) were significant at the $P < 0.05$ level before correcting for multiple testing. Adding simulated environmental variation $(V_E = V_A/10)$ and $V_E = V_A/2$) to trait measurements increased the variance of c_m, with this excess variance falling disproportionately along the later PCs (those that explain less variation in relatedness). These results suggest that unaccounted V_E increases estimated variance at later PCs, reducing power to detect the selective signal of increased variance at earlier PCs relative to later PCs. However, this reduction in power can be minimized by controlling environmental noise—for example by measuring line replicates in a common garden or best unbiased linear predictions (BLUPs) from multiple environments (See Appendix A for a more extensive treatment of V_E).

We tested for selection along 22 PCs for 22 traits that, themselves, are estimates of the breeding value (BLUPs) of these traits measured across multiple environments (Hung *et al.* 2012). These 22 traits include a number of traits thought to be important for adaptation to domestication and/or temperate environments in maize, such as flowering time (Swarts *et al.* 2017), upper leaf angle (Duvick 2005), and plant height (Duvick 2005; Peiffer *et al.* 2014). After controlling for multiple testing using a false discovery rate (FDR) of 0.05, we found evidence of adaptive divergence for three traits: days to silk, days to anthesis, and node number below ear (Figure 3A). These results are generally robust to the choice of PCs used for the denominator of Equation 8
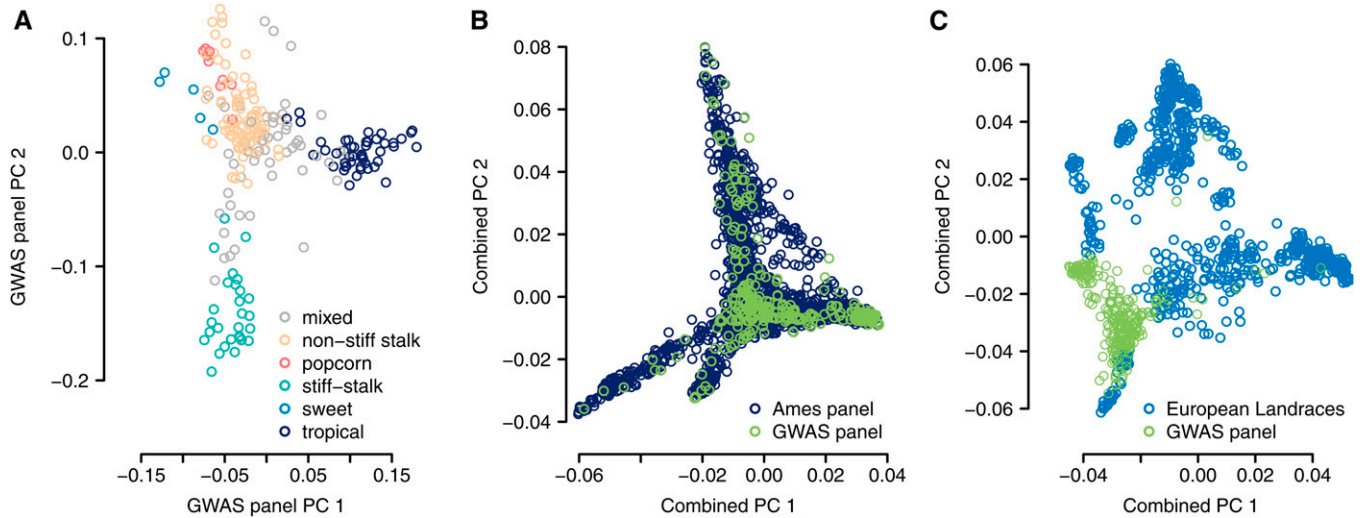
Figure 2 Structure in the maize populations. These plots show the first two PCs of population structure (the eigenvectors of the kinship matrix) for various maize panels included in this paper. (A) 240 maize lines from the "GWAS panel" that were used in the trait Q_{PC} analysis. Each point represents an inbred line and points are colored by their assignment to subpopulations from Flint-Garcia *et al.* (2005). (B) The GWAS panel from (A) along with the 2704 inbred maize lines of the Ames diversity panel (Romay *et al.* 2013). (C) The GWAS panel from (A) along with 906 European maize landraces from Unterseer *et al.* (2016).

used to estimate V_A (Supplemental Material, Figure S3). We plot the relationship between PC1 and two example traits to illustrate the data underlying these signals of selection. In Figure 3B, we show a relationship between PC1 and Kernel Number that is consistent with neutral processes, and, in Figure 3C, we show a relationship between PC 1 and days to silk that is stronger than would be expected due to neutral processes and is instead consistent with diversifying selection. We detected evidence of diversifying selection on various traits along PC1, PC2, and PC10. While PC 1 and PC2 differentiate between known maize subpopulations (Figure 2A), PC 10 separates out individuals within the tropical subpopulation, so our results are consistent with adaptive divergence contributing to trait variation within the tropical subpopulation (Figure S2).

Detecting selection in unphenotyped individuals using polygenic scores

Extending the method described above to detect selection in individuals or lines that have been genotyped but not phenotyped will allow the study of polygenic adaptation when phenotyping is expensive or impossible. Here, we outline methods for detecting selection in individuals that have been genotyped but not phenotyped (we refer to these individuals as the "genotyping panel"). We build on methods developed in Berg and Coop (2014) and Berg *et al.* (2017) and extend them to test for adaptive divergence along specific PCs and in the presence of population structure shared between the GWAS panel and the genotyping panel. The main difference between this test and Q_{PC} as described previously is that we now test for selection on polygenic scores, not on measured phenotypes. Specifically, if we have a set of n independent, trait-associated loci found in a GWAS, we can write the polygenic score for individual or line i as:

$$X_i = \sum_{j=1}^{n} 2\beta_i p_{ij}, \qquad (9)$$

where β_j is the additive effect of having an alternate allele of the jth locus, and p_{ij} is the alternate allele's frequency within the ith individual or line (*i.e.*, half the number of allele copies in a diploid individual).

However, when there is shared population structure between the GWAS panel and the genotyping panel, there are two concerns about testing for selection on polygenic scores made for the genotyping panel:

1. If we have already found a signal of selection on our phenotypes of interest in the GWAS panel, then a significant test could simply reflect this same signal and not independent adaptation in the genotyping panel.
2. Controls for population structure in the GWAS could bias the loci identified and the effect sizes estimated for these loci, leading to false positive signals of selection in the genotyping panel.

Modern GWAS control for false-positive associations due to population structure by incorporating a random effect based on the kinship matrix into the GWAS model (Yu *et al.* 2006). However, this approach will bias GWAS toward finding associations at alleles whose distributions do not follow neutral population structure, and toward missing true associations with loci whose distributions do follow population structure (Atwell *et al.* 2010). Because of this bias, the loci detected may not appear to have neutral distributions in the GWAS panel, or, crucially, in any additional set of populations that share structure with the GWAS panel.

We illustrate the problem by looking at the relationship between PC 1 and polygenic scores for a trait that has been
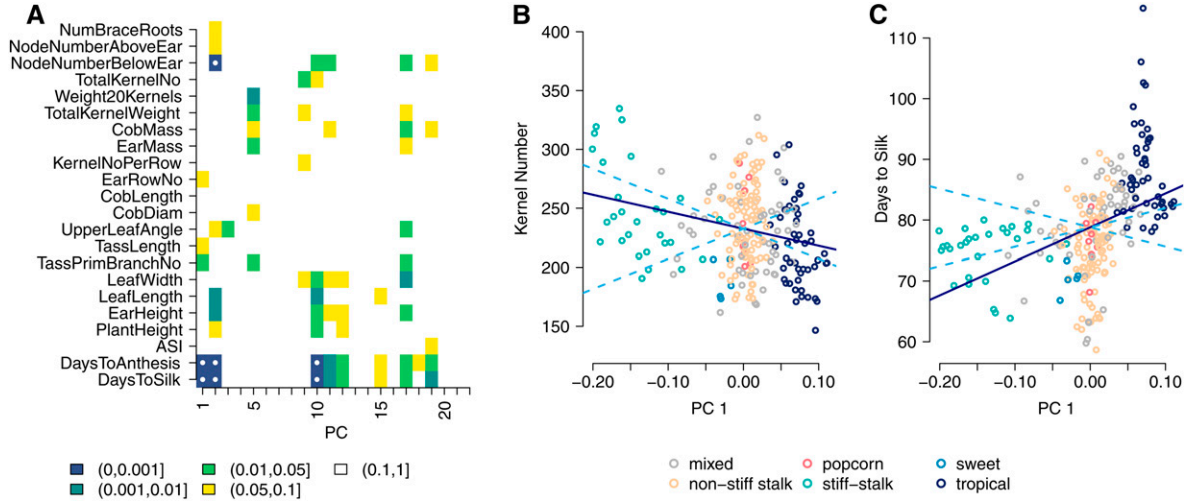
Figure 3 Detecting adaptation within the GWAS panel with Q_{PC}. (A) A heatmap showing results from Q_{PC} on the first 22 PCs (x-axis) for 22 traits (y-axis). Squares are colored by their P value. If the FDR ("q value") corresponding to that P value is < 0.05 there is a white dot in the square. (B) Total kernel number per cob plotted against the first PC of relatedness (PC 1). Each point represents a line in the GWAS panel, colored by its membership in a subpopulation (same colors as Figure 2A). The solid line shows the linear regression of the trait on PC 1 and the dashed lines show the 95% confidence interval of linear regressions expected under neutrality. Note that the linear regression is not the same as the F test done in Q_{PC}, and that we plot these lines for visualization purposes only. (C) Similar to (B), but showing days to silk on the Y axis. The regression line for days to silk against PC 1 (solid line) falls outside the expectation based on neutrality (dotted line), consistent with selection increasing trait divergence across PC 1.

simulated to be evolving neutrally in the GWAS panel and a set of related maize lines (Figure 4A). While there is no strong relationship between the simulated trait and PC 1 (Figure 4A), the polygenic scores calculated from GWAS results do show a strong correlation with PC 1 (Figure 4B). Note that the simulated trait values and polygenic scores have been standardized by dividing by the square root of their respective V_As estimated from the lower PCs, so this erroneous correlation not simply a result of the inflation of effect sizes in the GWAS due to false positives and the winner's curse, but represents an issue caused by shared population structure between the GWAS panel and the genotyping panel. The magnitude of the slope of the relationship between standardized polygenic score and PC 1 was greater than the magnitude of the slope of the relationship between standardized simulated trait and PC 1 for 162 out of 200 simulations.

Here, we control for shared structure between the GWAS and genotyping panels by conditioning on the estimated polygenic scores in the GWAS panel ($\overrightarrow{X_2}$) when assessing patterns of selection on the polygenic scores of a genotyping panel ($\overrightarrow{X_1}$). Specifically, following the multivariate normality assumption (Equation 1), we model the combined vector of polygenic scores in both panels as:

$$\begin{pmatrix} \overrightarrow{X_1} \\ \overrightarrow{X_2} \end{pmatrix} \sim MVN\left(\begin{pmatrix} \mu \\ \mu \end{pmatrix}, V_A \begin{pmatrix} K_{11} & K_{12} \\ K_{12}^T & K_{22} \end{pmatrix} \right). \quad (10)$$

where μ is the mean of the combined vector (X_1, X_2), K_{11} and K_{22} are the kinship matrices of the genotyping and GWAS panels, and K_{12} is the set of relatedness coefficients between lines in the genotyping panel (rows) and GWAS panel (columns). Note that the combination of the four kinship matrices in the variance term of Equation 10 is equivalent to the

kinship matrix of all individuals in the genotyping and GWAS panels. We discuss the mean centering of these matrices in Appendix C.

The conditional multivariate null model for polygenic scores in the genotyping panel conditional on the GWAS panel is then

$$\overrightarrow{X_1} \Big| \overrightarrow{X_2} \sim MVN\left(\overrightarrow{\mu}', 2V_a K' \right), \quad (11)$$

where $\overrightarrow{\mu}'$ is a vector of conditional means with an entry for each sample in the genotyping panel,

$$\overrightarrow{\mu'} = \mu + K_{12} K_{22}^{-1} (\overrightarrow{X_2} - \mu) \quad (12)$$

and K' is the relatedness matrix for the genotyping panel conditional on the matrix of the GWAS panel,

$$K' = K_{11} - K_{12} K_{22}^{-1} K_{12}^T. \quad (13)$$

Following Equation 6 and Equation 8, we can test for excess variation along the PCs of K', using the difference between polygenic scores $\overrightarrow{X_1}$ and the conditional means $\overrightarrow{\mu}'$ as the phenotype. Specifically, if $\overrightarrow{U}_m'$ and λ_m' are the mth eigenvector and eigenvalue of K', then

$$C_m' = \frac{\left(\overrightarrow{X} - \overrightarrow{\mu}' \right) \overrightarrow{U}_m'}{\sqrt{\lambda_m}'}, C_m' \sim N(0, 2V_A) \quad (14)$$

and

$$Q_{PC} = \frac{\mathbb{E}[C_m'^2]}{\mathbb{E}[\overrightarrow{C}_{R:M}'^2]} \sim F(1, M - R) \quad (15)$$
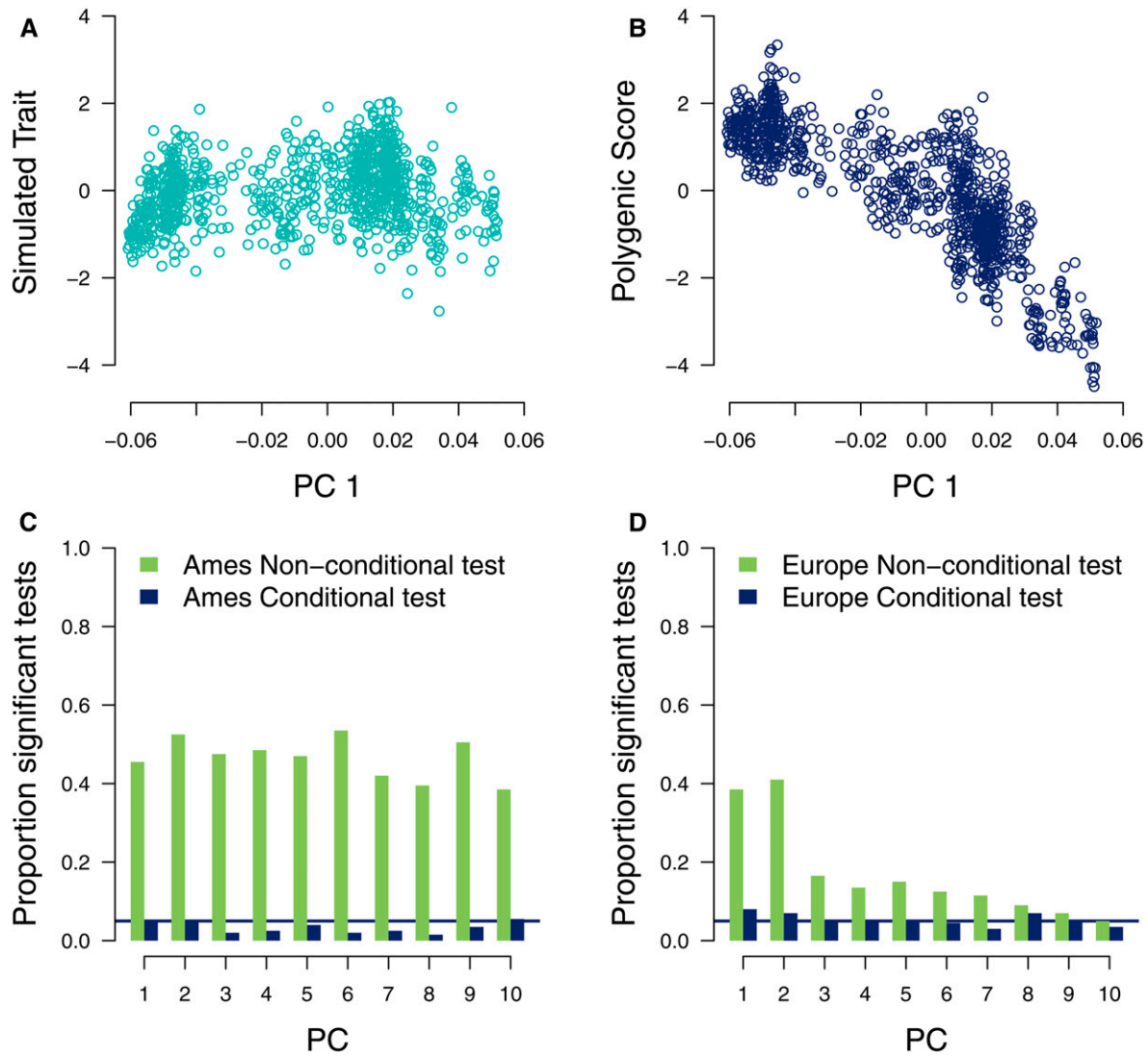
Figure 4 Simulations of Q_{PC} on polygenic scores. (A) The relationship between a neutrally simulated trait and PC 1 in the European landraces. Each point represents one individual and the trait value has been divided by $\sqrt{V_A}$ so that it can be compared to traits with different values of V_A. (B) The relationship between polygenic scores calculated for the same trait as in (A), based on a GWAS in the GWAS panel. This relationship between polygenic scores and PC is what is used in the nonconditional Q_{PC} test. As in (A), each point represents one maize individual and trait has been standardized by $\sqrt{V_A}$. (C) The proportion of 200 neutral simulations that were significant at the $P < 0.05$ level for the nonconditional Q_{PC} test and the conditional Q_{PC} test. A horizontal line is plotted at 0.05, to show the proportion of significant tests expected under the null hypothesis. (D) The same information, this time for the European landraces.

where $R > m$ and $M > R$. We will refer to the conditional version of the test as "conditional Q_{PC}".

It is worth taking some time to discuss how the conditional test controls for the two issues due to shared structure. First, by incorporating the polygenic scores of individuals in the GWAS panel into the null distribution of conditional Q_{PC}, we are able to test directly for adaptive divergence that occurred in the genotyping panel. Berg *et al.* (2017) also use the conditional test in this manner. Second, the conditional test forces the polygenic scores of individuals in the genotyping panel into the same multivariate normal distribution as the polygenic scores of individuals in the GWAS panel. The polygenic scores of GWAS individuals will include the ascertainment biases expected due to controls for structure in the GWAS, so ascer-

tainment bias will be incorporated into the null distribution of polygenic scores expected under drift. Now, we will detect selection only if trait divergence exceeds neutral expectations based on this combined multivariate normal distribution.

Applying Q_{PC} to polygenic scores in North American inbred maize lines and European landraces

First, we conducted a set of neutral simulations to assess the ability of the conditional Q_{PC} test to control for false positives due to shared structure. We applied both the conditional and original ("nonconditional") Q_{PC} test to detect selection on polygenic scores constructed from simulated neutral loci in two panels of maize genotypes that have not been extensively phenotyped: a set of 2815 inbred lines from the United States

Department of Agriculture (USDA) that we refer to as "the Ames panel" (Romay *et al.* 2013), and a set of 906 individuals from 38 European landraces (Unterseer *et al.* 2016). We chose these two panels to evaluate the potential of conditional Q_{PC} to control for shared population structure when the problem is severe, as in the Ames panel (Figure 2B), and moderate, as in the European landraces (Figure 2C). In addition, we expect that the evolution of many quantitative traits has been important for European landraces as they adapted to new European environments in the last ∼500 years (Tenaillon and Charcosset 2011; Unterseer *et al.* 2016).

False positive signatures of selection were common when using the original, nonconditional Q_{PC} to test for selection on polygenic scores based on loci simulated under neutral processes (Figure 4, C and D and Figure S4) even though the GWAS used to identify the loci used to make polygenic scores both had low power to detect associations and had high false positive rates. (Figure S5). The increase in false positive signals of selection due to shared structure persisted to much later PCs in the Ames panel than in the European landraces, likely because the extent of shared structure is more pervasive for the Ames panel. However, the conditional Q_{PC} test appeared to control for false positives in both the Ames panel and the European landraces (Figure 4, A and B).

We then conducted GWAS on 22 traits in the GWAS panel, using a *P* value cutoff of 0.005. This cutoff is less stringent then the cutoffs standardly used in maize GWAS (Romay *et al.* 2013; Peiffer *et al.* 2014), but allowed us to detect a large number of loci that we could use to construct polygenic scores. After thinning the loci for linkage disequilibrium (LD), we found associations for all traits with an average of 350 associated SNPs per trait (range 254–493). We used these SNPs to construct polygenic scores for lines in the Ames panel and individuals in the European landraces following Equation 9.

When we applied the original (nonconditional) test from Equation 8 to polygenic scores for 22 traits in the Ames panel on 182 PCs (4004 tests total), we detected widespread polygenic adaptation (Figure S6A and Figure S7A). In contrast, conditional Q_{PC} on the same set of traits and PCs found no signatures of polygenic adaptation in the Ames panel that survived control for multiple testing (Figure S6B and Figure S7B). The lack of results in the conditional test is unsurprising because the GWAS panel's population structure almost completely overlaps the Ames panel (Figure 2B), so once variation in the GWAS panel is accounted for in the conditional test, there is likely little differentiation in polygenic scores left to test for selection. We report these results to highlight the caution that researchers should use when applying methods for detecting polygenic adaptation to genotyping panels that share population structure with GWAS panels.

In the European landraces, we conducted Q_{PC} on 22 traits across 17 PCs (374 tests total), and, while we detected selection on a number of traits, as with the Ames panel, none of these signals were robust to controlling for multiple testing using a FDR approach (Figure S7D). However, we report the

results that were significant at an uncorrected level in Figure 5A to demonstrate how these types of selective signals could be visualized with these approaches. In Figure 5B, we show the relationship between conditional PC1 ($\vec{U}_m'$) and the difference between polygenic score for the number of brace roots and a conditional expectation ($\vec{X} - \vec{u}'$), which was our strongest signal of selection in the panel.

We conducted power simulations by shifting allele frequencies of GWAS-identified loci along a latitudinal selective gradient in the European landraces (see *Materials and Methods* section for details). When selection was strong (selection gradient $\alpha = 0.05$), we detected signals of selection in all 200 simulations along the first conditional PC, which had the strongest association with latitude. When selection was moderate ($\alpha = 0.01$) we detected selection in 57 of 200 simulations (Figure 5, C and D). These results suggest that there is power to detect selection on polygenic scores with Q_{PC} in the European landraces if selection actually occurs on the loci used to make these polygenic scores.

Discussion

In this paper, we have laid out a set of approaches that can be used to study adaptation and divergent selection using genomic and phenotypic data from structured populations. We first described a method, Q_{PC}, that can be used to detect adaptive trait divergence in a species-wide sample of individuals or lines that have been phenotyped in common garden and genotyped. We demonstrated this method using a panel of phenotyped domesticated maize lines, showing evidence of selection on flowering time and node number below ear. Second, we present an extension of Q_{PC} that can be applied to individuals related to the GWAS panel that have not themselves been phenotyped using a conditional test to avoid confounding due to shared population structure. We showed that this test is robust to false positives due to population structure shared between the GWAS panel and the genotyping panel, and that it has power to detect selection. We applied this method to two panels of maize lines and showed marginal evidence of selection on a number of traits, but these signals were not robust to multiple testing corrections. Overall, the methods described and demonstrated here will be useful to a wide range of study systems.

While we were able to use Q_{PC} to detect diversifying selection on phenotypes in the GWAS panel of 240 inbred lines, we were unable to detect similar patterns using polygenic scores for the Ames panel and European landraces (after controlling for multiple testing). This lack of selective signal was expected in Ames because the high overlap in relatedness between the Ames panel and the GWAS panel reduces power to detect selection in the Ames panel alone. However, our simulations showed that we did have power to detect moderate to strong selection acting on GWAS-associated loci in European landraces, and we expect that adaptation to European environments has contributed to trait diversification (Unterseer *et al.* 2016). There are a few factors that
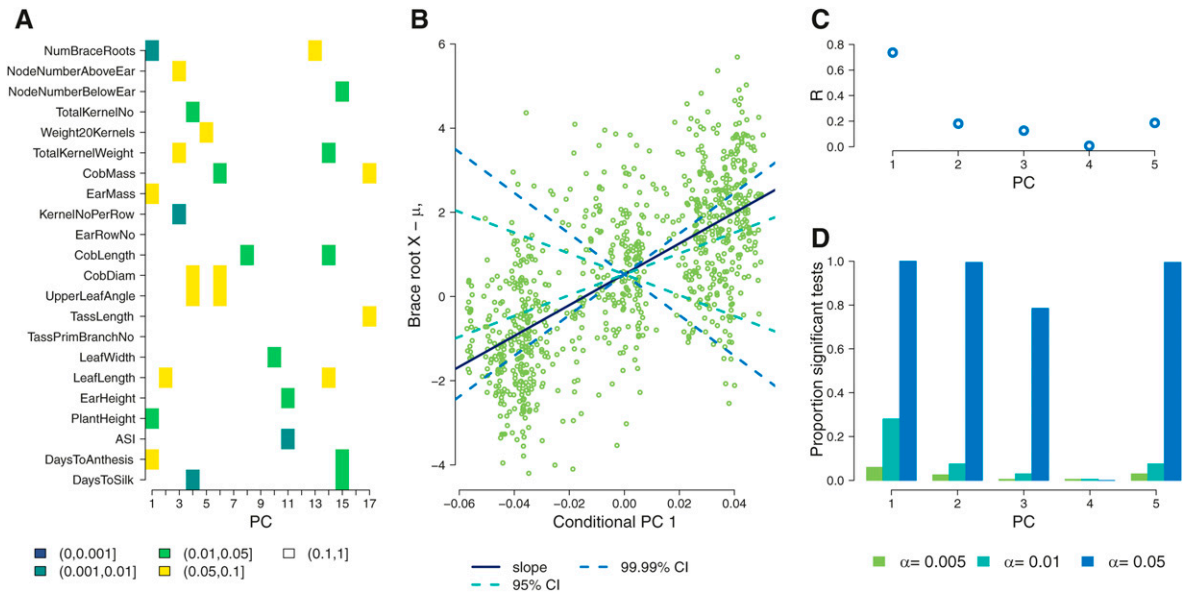
Figure 5 Detecting adaptation within the European landraces with Q_{PC} on polygenic scores. (A) A heatmap showing results from the conditional Q_{PC} on the first 17 PCs (x-axis) for 22 traits (y-axis). Squares are colored by uncorrected P value if $P < 0.1$. (B) Marginal evidence of selection on brace root number. The difference between polygenic score for brace root and the conditional mean ($\overrightarrow{X} - \overrightarrow{\mu}'$) is plotted against the conditional PC 1 for each line. The solid line shows the observed linear relationship between conditional PC 1 and $\overrightarrow{X} - \overrightarrow{\mu}'$ and the dotted lines show the 95% confidence interval for this slope based on neutral expectations uncorrected for multiple testing and the 99.99% confidence interval corresponding to a Bonferonni correction for multiple testing. (C) The absolute value of the correlation coefficient (R) between PCs and latitude. (D) The proportion of significant tests for traits simulated to be under selection along latitude for three different selection strengths.

could explain our inability to detect selection on polygenic scores for European landraces. First, the polygenic scores we constructed used GWAS results from traits measured in North American environments. If there is $G \times E$ for these traits, we may not be measuring traits that are actually under selection in Europe. Second, it is likely that in our small GWAS panel (n = 263 or 281) we are underpowered to detect most causal loci and so our predictions are too inaccurate to pull out a signal of selection. All together, our results suggest that while GWAS are undoubtedly useful to identify loci underlying traits, an analysis of phenotypes expressed in a common environment will often be the most powerful approach for detecting adaptation, especially in systems with underpowered GWAS.

Our proposed methods use PCA to separate out independent axes of population structure. There is a clear connection between PCA and average pairwise coalescent times (McVean 2009), and, because of this connection, PCA has been useful in a range of population genetic applications, including the detection and visualization of population structure (Patterson *et al.* 2006; Novembre *et al.* 2008), understanding the roles of population history and geography (Menozzi *et al.* 1978; Novembre and Stephens 2008), controlling for population structure in GWAS (Price *et al.* 2006). Here we use PCs to define broad axes of variation analogous to the between-population variation described in standard $Q_{ST} - F_{ST}$ tests while using later PCs to describe variation within populations. In our application to maize data, we use arbitrary cutoffs to define which PCs represent

between- and within-population variation. In the future, developing a rigorous method for choosing appropriate PCs will be useful, but, for now, we suggest that researchers use their own knowledge of their study system and datasets to choose these PCs. We also caution that the presence of many close relatives in the sample could affect PCA, such that early PCs could end up representing variation within a large family, and we suggest that researchers remove close relatives when using these methods. In addition, while PCs provide a useful way of separating signals, in some cases the constraints of PCA make the PCs unintuitive in terms of geography and environmental variables (Novembre and Stephens 2008). Therefore, it will also be useful to explore approaches like that outlined in Equation 9 of Berg *et al.* (2018) that project trait values or polygenic scores onto environmental variables to test if the relationship between trait and environment is larger than would be expected due to drift.

There are a number of connections between the methods presented here and previous approaches. Ovaskainen *et al.* (2011), Karhunen *et al.* (2013) calculated a $Q_{ST} - F_{ST}$ -like measure of diversifying selection using the kinship matrix to model variation in relatedness among subpopulations. Their approach, however, is still reliant on identifying subpopulations and on using trait measurements in families or crosses to obtain estimates of V_A. For single loci, a number of F_{ST}-like approaches have been developed that use PCs to replace subpopulation structure to detecting individual outlier loci that deviate from a neutral model of population structure

(Duforet-Frebourg *et al.* 2015; Chen *et al.* 2016; Galinsky *et al.* 2016; Luu *et al.* 2017). Our methods can be viewed as a a phenotypic equivalent to these locus-level approaches. In addition, Liu *et al.* (2018) have recently explored a related approach using projections of polygenic scores along PCs. Finally it may be useful to recast our method in terms of the animal model by splitting the kinship matrix into a "between population" matrix described by early PCs and a "within population" matrix described by later PCs. We could then detect selection by comparing estimates of V_A for these two matrices. Such an animal-model approach may also offer a way to incorporate environmental variance in systems where replicates of the same genotype are not possible.

There are a number of caveats for applying Q_{PC} to traits on additional systems and datasets that stem from issues in accurately estimating V_A, and it is important to carefully consider the assumption underlying Q_{PC} that all traits are made up of additive combinations of allelic effects. If environmental variation contributes to trait variation, it will reduce the power of Q_{PC} to detect diversifying selection because environmental variation will contribute most to variation at later PCs (Appendix A). Additionally, additive-by-additive epistasis has the potential to contribute to false-positive signals of adaptation because additive-by-additive epistatic variation will contribute most to phenotypic variance along earlier PCs (Appendix B). In general, nonadditive interactions between alleles may cause difficulty for Q_{PC} in systems, like maize, where traits are measured on inbred lines but selection occurs on outbred individuals. However, there is evidence that additive-by-additive variance will often be small compared to V_A within populations (Hill *et al.* 2008); for example, the genetic basis of flowering time variation in maize is largely additive (Buckler *et al.* 2009), suggesting that our conclusions about adaptive divergence in flowering time are likely robust to concerns about epistasis.

Our results highlight a number of issues with polygenic adaptation tests that depend on polygenic scores using GWAS-associated loci. As has been recently highlighted by Berg *et al.* (2018) and Sohail *et al.* (2018), structure in a GWAS panel can contribute to false signals of polygenic adaptation in polygenic scores constructed from the results of that panel. We observed that this problem is especially strong when there is shared population structure between the GWAS panel and the genotyping panel used to construct polygenic scores but that the use of a conditional test that accounts for shared structure between the two datasets can control for these false positives. There is potential for these methods to be used to address problems due to structure in GWAS panels in both nonhuman and human systems, although the conditional test approach would need to be adapted to the very large sample sizes used in human GWAS.

All together, the methods presented here provide an approach to detecting the role of diversifying selection in shaping patterns of trait variation across a number of species and traits. A number of further avenues exist for extending these methods. First, we applied this test to traits independently, but

extending Q_{PC} to incorporate multiple correlated traits will likely improve power to detect selection by reducing the number of tests done. In addition, this extension could allow the detection of adaptive changes in trait correlations. Second, these methods could be extended to take advantage of more sophisticated methods of genomic prediction than the additive model presented here (as in Beissinger *et al.* 2018; Liu *et al.* 2018). Pursuing this goal will require carefully addressing issues related to LD between marker loci. Overall, developing and applying methods for detecting polygenic adaptation in a wide range of species will be crucial for understanding the broad contribution of adaptation to phenotypic divergence.

Materials and Methods

Analyses were done in R and we used the dplyr package (Wickham *et al.* 2017; R Core Team 2018). All code is available at https://github.com/emjosephs/qpc-maize.

The germplasm used in this study

We analyzed three different maize diversity panels.

The GWAS panel: The Major Goodman GWAS panel, also sometimes referred to as "the 282" or "the Flint Garcia GWAS Panel," contains 302 inbred lines meant to represent the genetic diversity of public maize-breeding programs (Flint-Garcia *et al.* 2005). Genotype-by-sequencing (GBS) data are available for 281 of these lines from Romay *et al.* (2013) and 7X genomic sequence from 271 of these lines is available from Bukowski *et al.* (2017). In addition, these lines have been phenotyped for 22 traits in multiple common garden experiments (Hung *et al.* 2012).

The Ames panel: A panel of 2815 inbred lines from the USDA that have been genotyped with GBS (Romay *et al.* 2013) at 717,588 SNPs.

The European landraces: A panel of 906 individuals from 38 European landraces (31 Flint-type and 7 Dent-type) that were genotyped at 547,412 SNPs using an array (Unterseer *et al.* 2016).

Q_{PC} in the GWAS panel

We tested for selection on 22 traits phenotyped in the GWAS panel. BLUPs for these traits were sourced from Hung *et al.* (2012) and genomic sequence data from Bukowski *et al.* (2017). Out of the 302 individuals in the GWAS panel, we retained 240 individuals that had data for all 22 traits of interest and had genotype calls for >70% of the SNPs in the genomic dataset.

To construct a kinship matrix for the GWAS panel, we randomly sampled 50,000 SNPs from across the genome after removing sites that were missing any data or had unrealistic levels of heterozygosity (the proportion of heterozygous individuals exceeded 0.5). The allele frequencies (0, 0.5, or 1) for individuals at these 50,000 SNPs were arranged in an *MxN*

matrix (referred to here as G), where M is the number of individuals (240) and N is the number of loci (50,000) such that g_{in} is the entry on the ith row and n^{th} column of G, and it represents the genotype of the ith individual at the nth locus.

Then we centered the matrix using a centering matrix, T, which is an $M-1$ by M matrix with $\frac{M-1}{M}$ on the diagonal and $\frac{-1}{M}$ at all other cells. The matrix TG will be the matrix G that has been mean centered, such that if $(tg)_{in}$ is the entry for the ith individual and n^{th} locus and $\overline{g_n}$ is the mean allele frequency of the nth locus, $(tg)_{in} = g_{in} - \overline{g_n}$. Note that multiplying G by T also drops one individual from the kinship matrix to reflect the fact that by mean centering, we have lost one degree of freedom. We also standardized G by dividing by the square root of the expected heterozygosity of all loci, calculated by taking the mean of $\epsilon(1-\epsilon)$ across all loci, where ϵ is the mean allele frequency of a locus. All together, we calculated K as the covariance of the centered and standardized matrix:

$$K = \text{cov}\left(\frac{TG}{\sqrt{\mathbb{E}[\epsilon(1-\epsilon)]}}\right) \tag{16}$$

Each cell of K contains the covariance of genotypes between individuals, such that if K_{ij} is the value of the ith row, and j^{th} column of K it represents the covariance between the ith and j^{th} individuals, such that,

$$K_{ij} = \frac{1}{\sqrt{\mathbb{E}[\epsilon(1-\epsilon)]}} \sum_{n=1}^{N} (tg)_{in}(tg)_{jn}$$

$$= \frac{1}{\sqrt{\mathbb{E}[\epsilon(1-\epsilon)]}} \sum_{n=1}^{N} (g_{in} - \overline{g_n})(g_{jn} - \overline{g_n}) \tag{17}$$

The principal components of the genotype matrix G were calculated from the eigenvectors of K. However, an equivalent way to estimate the PCs would be to conduct a singular value decomposition on a mean centered G (McVean 2009). We used the kinship matrix K to test for selection on traits using Equation 8 on the first 22 PCs that, cumulatively, explain 30% of the variation in K.

For the denominator of Equation 8 we used the last half of the PCs (119–238). We plotted 95% confidence intervals for the mth PC for display purposes as lines with the same mean as the observed data and a slope of $\pm 1.96 * \sqrt{\lambda_m \mathbb{E}[C^2_{119:238}]}$. We conducted 200 simulations by simulating traits that evolve neutrally along the kinship matrix using the mvrnorm R function in the MASS package (Venables and Ripley 2002) and testing for selection using Equation 8.

GWAS in maize inbreds

We used GEMMA (Zhou and Stephens 2012) to conduct GWAS for trait BLUPs in the GWAS panel, controlling for population structure with a standardized kinship matrix generated by GEMMA (note that this matrix is different from the one used in tests of selection with Q_{PC}). We conducted two separate GWAS for use in testing for selection in the Ames

panel and in the European panel separately. First, for finding SNP associations that we could use to construct breeding values in the Ames panel, we used GBS data for 281 lines from the GWAS panel that had been genotyped by Romay et al. (2013). Next, for finding SNP associations for constructing breeding values in the European landraces, we took whole genome data from Bukowski et al. (2017) for 263 individuals that had genotype calls for $>70\%$ of polymorphic sites and extracted genotypes for sites that overlapped with those present in the European landrace dataset from Unterseer et al. (2016). All genotypic data were aligned to v3 of the maize reference genome, except the genotypes of the European landraces, which we lifted over from v2 to v3 using CrossMap (Zhao et al. 2013). For both sets of GWAS analyses, we tested all SNPs with a minor allele frequency above 0.01, <0.05 missing data, and we picked all hits with a likelihood-ratio test P value below 0.005. We pruned both sets of SNPs by using a linkage map from Ogut et al. (2015) to construct one cM windows with GenomicRanges in R (Lawrence et al. 2013). We picked the SNP with the lowest P value per window and, when multiple SNPs had the same P value, we sampled one SNP randomly.

Simulated populations and traits

We generated data for Figure 1 using ms to do coalescent simulations (Hudson 2002) for three large populations of 60 individuals, each with two smaller subpopulations of 30 individuals. Simulations were conducted with the following command: ms 180 1 -t 500 -r 500 10000 -I 6 30 30 30 30 30 30 -ej 0.06 1 2 -ej 0.06 3 4 -ej 0.06 5 6 -ej 0.1 4 6 -ej 0.15 2 6. This command yielded 6689 loci polymorphic loci for 180 individuals. Neutrally evolving trait values were determined by summing up the number of nonreference allele copies each individual has, multiplied by these effect sizes for 50 randomly chosen causal loci (as in Equation 9). The other 6639 loci were used to generate a kinship matrix following Equation 16 and the eigendecomposition of this matrix provided principal components.

Applying Q_{PC} to polygenic scores from the Ames panel and European landraces

We generated combined genetic matrices for each genotyping panel (either Ames or European) and the GWAS panel. In both datasets, we removed sites with a minor allele frequency below 0.01 and a proportion of missing data > 0.05 across the combined dataset, leaving 108,110 SNPs in the Ames-GWAS dataset and 441,986 SNPs in the European-GWAS dataset. Missing data points were imputed by replacing each missing genotype from a random sample of the pool of genotypes present in the individuals without missing data. The random imputation step was done once for each missing data point and the same randomly imputed dataset was used for all subsequent analyses.

We constructed kinship matrices for the Ames panel combined with the GWAS panel and the European panel combined with the GWAS panel following the procedure described in

Equation 16, using 50,000 randomly sampled SNPs with a minor allele frequency > 0.01 and a proportion of missing data < 0.05. The genotype information from the combined datasets was used to construct polygenic scores following Equation 9.

We used these polygenic scores to test for diversifying selection on 22 traits (described above) for the PCs that cumulatively explained the first 30% of variation in the conditional kinship matrix (182 for the Ames panel and 17 for European maize). As in the Q_{PC} test, we chose the later 50% of PCs for the denominator. We used Qvalue (Storey *et al.* 2015) to generate FDR estimates ("q values").

Simulations of Q_{PC} on polygenic scores

We conducted neutral simulations to detect the rate of false-positive inferences of selection on neutrally evolving traits. We conducted 800 total: 400 with the Ames panel and 400 with the European landrace panel, and within each panel, 200 with 500 causal loci and 200 with 50 causal loci. For each simulation, we simulated a phenotype by randomly picking 500 or 50 sites in the combined genotype datasets and assigning each alternate allele an effect size drawn from a normal distribution with mean 0 and variance 1. For each individual in the GWAS panel, we then calculated a simulated breeding value following Equation 9. These simulated traits were mapped using a GWAS with the same procedure described above. These simulations showed the GWAS in our sample had relatively low power to detect causal alleles, especially ones of small effect, and also identified a large number of false-positives, as could be expected for a GWAS of this size (Figure S5).

The loci identified in these GWAS ($P < 0.005$) were pruned for LD and then used for analysis. We tested for evidence of diversifying selection on polygenic scores in two ways for each set of simulations. First, we used Q_{PC} with the standard kinship matrix generated using lines in the genotyping set (either Ames or European landraces). Second, we used the conditional Q_{PC} test described in Equation 11.

We also conducted power simulations using the European landraces. We first simulated traits evolving under diversifying selection by taking trait-associated loci from the neutral simulations and shifting the allele frequencies at these loci in the European landraces based on their latitude of origin. Let p be the initial allele frequency in the jth landrace population, L the latitude of the jth landrace population, β the effect size of a alternate allele at the ith locus, ϵ the mean allele frequency of the ith allele, α the selection gradient, and p' the allele frequency after selection. Then

$$p' = p + \beta\epsilon(1 - \epsilon)\alpha L \qquad (18)$$

We conducted simulations for three values of α: 0.05, 0.01, and 0.005 and tested for selection with conditional Q_{PC}.

Data availability

All code and data are available at https://github.com/emjosephs/qpc-maize. Supplemental material available at Figshare: https://doi.org/10.25386/genetics.7607471.

Literature Cited

Aitken, S. N., S. Yeaman, J. A. Holliday, T. Wang, and S. Curtis-McLane, 2008 Adaptation, migration or extirpation: climate change outcomes for tree populations. Evol. Appl. 1: 95–111. https://doi.org/10.1111/j.1752-4571.2007.00013.x

Atwell, S., Y. S. Huang, B. J. Vilhjálmsson, G. Willems, M. Horton *et al.*, 2010 Genome-wide association study of 107 phenotypes in *Arabidopsis thaliana* inbred lines. Nature 465: 627–631. https://doi.org/10.1038/nature08800

Bay, R. A., N. Rose, R. Barrett, L. Bernatchez, C. K. Ghalambor *et al.*, 2017 Predicting responses to contemporary environmental change using evolutionary response architectures. Am. Nat. 189: 463–473. https://doi.org/10.1086/691233

Beissinger, T., J. Kruppa, D. Cavero, N.-T. Ha, M. Erbe *et al.*, 2018 A simple test identifies selection on complex traits. Genetics 209: 321–333. https://doi.org/10.1534/genetics.118.300857

Berg, J. J., and G. Coop, 2014 A population genetic signal of polygenic adaptation. PLoS Genet. 10: e1004412. https://doi.org/10.1371/journal.pgen.1004412

Berg, J. J, X. Zhang, and G. Coop, 2017 Polygenic adaptation has impacted multiple anthropometric traits. bioRxiv.

Berg, J. J., A. Harpak, N. Sinnott-Armstrong, A. M. Joergensen, H. Mostafavi *et al.*, 2018 Reduced signal for polygenic adaptation of height in UK Biobank. bioRxiv: 354951.

Brommer, J., 2011 Whither Pst? the approximation of Qst by Pst in evolutionary and conservation biology. J. Evol. Biol. 24: 1160–1168. https://doi.org/10.1111/j.1420-9101.2011.02268.x

Bryc, K., W. Bryc, and J. W. Silverstein, 2013 Separation of the largest eigenvalues in eigenanalysis of genotype data from discrete subpopulations. Theor. Popul. Biol. 89: 34–43. https://doi.org/10.1016/j.tpb.2013.08.004

Buckler, E. S., J. B. Holland, P. J. Bradbury, C. B. Acharya, P. J. Brown *et al.*, 2009 The genetic architecture of maize flowering time. Science 325: 714–718. https://doi.org/10.1126/science.1174276

Bukowski, R., X. Guo, Y. Lu, C. Zou, B. He *et al.*, 2017 Construction of the third-generation *Zea mays* haplotype map. Gigascience 7: gix134.

Chen, G.-B., S. H. Lee, Z.-X. Zhu, B. Benyamin, and M. R. Robinson, 2016 EigenGWAS: finding loci under selection through genome-wide association studies of eigenvectors in structured populations. Heredity 117: 51–61. https://doi.org/10.1038/hdy.2016.25

Duforet-Frebourg, N., K. Luu, G. Laval, E. Bazin, and M. G. Blum, 2015 Detecting genomic signatures of natural selection with principal component analysis: application to the 1000 genomes

data. Mol. Biol. Evol. 33: 1082–1093. https://doi.org/10.1093/molbev/msv334

Duvick, D., 2005   Genetic progress in yield of United States maize (*Zea mays L.*). Maydica 50: 193.

Falconer, D., and T. Mackay, 1996   *Introduction to Quantitative Genetics*. Longman, London.

Field, Y., E. A. Boyle, N. Telis, Z. Gao, K. J. Gaulton *et al.*, 2016   Detection of human adaptation during the past 2000 years. Science 354: 760–764.

Flint-Garcia, S. A., A.-C. Thuillet, J. Yu, G. Pressoir, S. M. Romero *et al.*, 2005   Maize association population: a high-resolution platform for quantitative trait locus dissection. Plant J. 44: 1054–1064. https://doi.org/10.1111/j.1365-313X.2005.02591.x

Galinsky, K. J., G. Bhatia, P.-R. Loh, S. Georgiev, S. Mukherjee *et al.*, 2016   Fast principal-component analysis reveals convergent evolution of ADH1B in Europe and East Asia. Am. J. Hum. Genet. 98: 456–472. https://doi.org/10.1016/j.ajhg.2015.12.022

Hadfield, J., and S. Nakagawa, 2010   General quantitative genetic methods for comparative biology: phylogenies, taxonomies and multi-trait models for continuous and categorical characters. J. Evol. Biol. 23: 494–508. https://doi.org/10.1111/j.1420-9101.2009.01915.x

Henderson, C. R., 1950   Estimation of genetic parameters. Biometrics 6: 186–187.

Henderson, C. R., 1953   Estimation of variance and covariance components. Biometrics 9: 226–252. https://doi.org/10.2307/3001853

Hereford, J., 2009   A quantitative survey of local adaptation and fitness trade-offs. Am. Nat. 173: 579–588. https://doi.org/10.1086/597611

Hill, W. G., 2010   Understanding and using quantitative genetic variation. Philos. Trans. R. Soc. Lond. B Biol. Sci. 365: 73–85. https://doi.org/10.1098/rstb.2009.0203

Hill, W. G., M. E. Goddard, and P. M. Visscher, 2008   Data and theory point to mainly additive genetic variance for complex traits. PLoS Genet. 4: e1000008. https://doi.org/10.1371/journal.pgen.1000008

Howden, S. M., J.-F. Soussana, F. N. Tubiello, N. Chhetri, M. Dunlop *et al.*, 2007   Adapting agriculture to climate change. Proc. Natl. Acad. Sci. USA 104: 19691–19696. https://doi.org/10.1073/pnas.0701890104

Hudson, R. R., 2002   Generating samples under a wright–fisher neutral model of genetic variation. Bioinformatics 18: 337–338. https://doi.org/10.1093/bioinformatics/18.2.337

Hung, H. Y., C. Browne, K. Guill, N. Coles, M. Eller *et al.*, 2012   The relationship between parental genetic or phenotypic divergence and progeny variation in the maize nested association mapping population. Heredity (Edinb) 108: 490–499. https://doi.org/10.1038/hdy.2011.103

Karhunen, M., J. Merilä, T. Leinonen, J. Cano, and O. Ovaskainen, 2013   DRIFTSEL: an R package for detecting signals of natural selection in quantitative traits. Mol. Ecol. Resour. 13: 746–754. https://doi.org/10.1111/1755-0998.12111

Kremer, A., and V. Le Corre, 2012   Decoupling of differentiation between traits and their underlying genes in response to divergent selection. Heredity (Edinb) 108: 375–385. https://doi.org/10.1038/hdy.2011.81

Latta, R. G., 1998   Differentiation of allelic frequencies at quantitative trait loci affecting locally adaptive traits. Am. Nat. 151: 283–292. https://doi.org/10.1086/286119

Lawrence, M., W. Huber, H. Pagès, P. Aboyoun, M. Carlson *et al.*, 2013   Software for computing and annotating genomic ranges. PLoS Comput. Biol. 9: e1003118. https://doi.org/10.1371/journal.pcbi.1003118

Le Corre, V., and A. Kremer, 2012   The genetic differentiation at quantitative trait loci under local adaptation. Mol. Ecol. 21: 1548–1566. https://doi.org/10.1111/j.1365-294X.2012.05479.x

Leimu, R., and M. Fischer, 2008   A meta-analysis of local adaptation in plants. PLoS One 3: e4010. https://doi.org/10.1371/journal.pone.0004010

Leinonen, T., R. B. O'Hara, J. M. Cano, and J. Merilä, 2008   Comparative studies of quantitative trait and neutral marker divergence: a meta-analysis. J. Evol. Biol. 21: 1–17. https://doi.org/10.1111/j.1420-9101.2007.01445.x

Leinonen, T., R. S. McCairns, R. B. O'Hara, and J. Merilä, 2013   Qst–Fst comparisons: evolutionary and ecological insights from genomic heterogeneity. Nat. Rev. Genet. 14: 179–190. https://doi.org/10.1038/nrg3395

Liu, X., P.-R. Loh, L. J. O'Connor, S. Gazal, A. Schoech *et al.*, 2018   Quantification of genetic components of population differentiation in UK Biobank traits reveals signals of polygenic selection. bioRxiv. doi: https://doi.org/10.1101/357483

Luu, K., E. Bazin, and M. G. Blum, 2017   pcadapt: an R package to perform genome scans for selection based on principal component analysis. Mol. Ecol. Resour. 17: 67–77. https://doi.org/10.1111/1755-0998.12592

McVean, G., 2009   A genealogical interpretation of principal components analysis. PLoS Genet. 5: e1000686. https://doi.org/10.1371/journal.pgen.1000686

Menozzi, P., A. Piazza, and L. Cavalli-Sforza, 1978   Synthetic maps of human gene frequencies in Europeans. Science 201: 786–792. https://doi.org/10.1126/science.356262

Mikel, M. A., and J. W. Dudley, 2006   Evolution of North American dent corn from public to proprietary germplasm. Crop Sci. 46: 1193–1205. https://doi.org/10.2135/cropsci2005.10-0371

Mitchell-Olds, T., J. H. Willis, and D. B. Goldstein, 2007   Which evolutionary processes influence natural genetic variation for phenotypic traits? Nat. Rev. Genet. 8: 845–856. https://doi.org/10.1038/nrg2207

Novembre, J., and N. H. Barton, 2018   Tread lightly interpreting polygenic tests of selection. Genetics 208: 1351–1355. https://doi.org/10.1534/genetics.118.300786

Novembre, J., and M. Stephens, 2008   Interpreting principal component analyses of spatial population genetic variation. Nat. Genet. 40: 646–649. https://doi.org/10.1038/ng.139

Novembre, J., T. Johnson, K. Bryc, Z. Kutalik, A. R. Boyko *et al.*, 2008   Genes mirror geography within Europe. Nature 456: 98–101 (erratum: Nature 456: 274). https://doi.org/10.1038/nature07331

Ogut, F., Y. Bian, P. J. Bradbury, and J. B. Holland, 2015   Joint-multiple family linkage analysis predicts within-family variation better than single-family analysis of the maize nested association mapping population. Heredity 114: 552–563. https://doi.org/10.1038/hdy.2014.123

Ovaskainen, O., M. Karhunen, C. Zheng, J. M. C. Arias, and J. Merilä, 2011   A new method to uncover signatures of divergent and stabilizing selection in quantitative traits. Genetics 189: 621–632. https://doi.org/10.1534/genetics.111.129387

Patterson, N., A. L. Price, and D. Reich, 2006   Population structure and eigenanalysis. PLoS Genet. 2: e190. https://doi.org/10.1371/journal.pgen.0020190

Peiffer, J. A., M. C. Romay, M. A. Gore, S. A. Flint-Garcia, Z. Zhang *et al.*, 2014   The genetic architecture of maize height. Genetics 196: 1337–1356. https://doi.org/10.1534/genetics.113.159152

Price, A. L., N. J. Patterson, R. M. Plenge, M. E. Weinblatt, N. A. Shadick *et al.*, 2006   Principal components analysis corrects for stratification in genome-wide association studies. Nat. Genet. 38: 904–909. https://doi.org/10.1038/ng1847

Prout, T., and J. Barker, 1993   F statistics in *Drosophila buzzatii*: selection, population size and inbreeding. Genetics 134: 369–375.

Pujol, B., A. J. Wilson, R. Ross, and J. Pannell, 2008   Are Qst–Fst comparisons for natural populations meaningful? Mol. Ecol. 17: 4782–4785. https://doi.org/10.1111/j.1365-294X.2008.03958.x

R Core Team, 2018   *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna.

Robinson, M. R., G. Hemani, C. Medina-Gomez, M. Mezzavilla, T. Esko *et al.*, 2015   Population genetic differentiation of height and body mass index across Europe. Nat. Genet. 47: 1357–1362. https://doi.org/10.1038/ng.3401

Romay, M. C., M. J. Millard, J. C. Glaubitz, J. A. Peiffer, K. L. Swarts *et al.*, 2013   Comprehensive genotyping of the USA national maize inbred seed bank. Genome Biol. 14: R55. https://doi.org/10.1186/gb-2013-14-6-r55

Savolainen, O., M. Lascoux, and J. Merilä, 2013   Ecological genomics of local adaptation. Nat. Rev. Genet. 14: 807–820. https://doi.org/10.1038/nrg3522

Sohail, M., R. M. Maier, A. Ganna, A. Bloemendal, A. R. Martin *et al.*, 2018   Signals of polygenic adaptation on height have been overestimated due to uncorrected population structure in genome-wide association studies. bioRxiv. doi: https://doi.org/10.1101/355057

Spitze, K., 1993   Population structure in *Daphnia obtusa*: quantitative genetic and allozymic variation. Genetics 135: 367–374.

Storey, J. D., A. J. Bass, A. Dabney, and D. Robinson, 2015   qvalue: Q-value estimation for false discovery rate control. R package version 2.8.0. https://cran.r-project.org/web/packages/qvalue/index.html

Swarts, K., R. M. Gutaker, B. Benz, M. Blake, R. Bukowski *et al.*, 2017   Genomic estimation of complex traits reveals ancient maize adaptation to temperate North. Am. Sci. 357: 512–515.

Takeda, S., and M. Matsuoka, 2008   Genetic approaches to crop improvement: responding to environmental and population changes. Nat. Rev. Genet. 9: 444–457. https://doi.org/10.1038/nrg2342

Tenaillon, M. I., and A. Charcosset, 2011   A European perspective on maize history. C. R. Biol. 334: 221–228. https://doi.org/10.1016/j.crvi.2010.12.015

Thompson, R., 2008   Estimation of quantitative genetic parameters. Proc. R. Soc. Lond. B Biol. Sci. 275: 679–686. https://doi.org/10.1098/rspb.2007.1417

Turchin, M. C., C. W. Chiang, C. D. Palmer, S. Sankararaman, D. Reich *et al.*, 2012   Evidence of widespread selection on standing variation in Europe at height-associated SNPs. Nat. Genet. 44: 1015–1019. https://doi.org/10.1038/ng.2368

Unterseer, S., S. D. Pophaly, R. Peis, P. Westermeier, M. Mayer *et al.*, 2016   A comprehensive study of the genomic differentiation between temperate dent and flint maize. Genome Biol. 17: 137. https://doi.org/10.1186/s13059-016-1009-x

Venables, W. N., and B. D. Ripley, 2002   *Modern Applied Statistics with S*, Ed. 4. Springer, New York. https://doi.org/10.1007/978-0-387-21706-2

Wang, W., R. Mauleon, Z. Hu, D. Chebotarov, S. Tai *et al.*, 2018   Genomic variation in 3,010 diverse accessions of Asian cultivated rice. Nature 557: 43–49. https://doi.org/10.1038/s41586-018-0063-9

Whitlock, M. C., 2008   Evolutionary inference from Qst. Mol. Ecol. 17: 1885–1896. https://doi.org/10.1111/j.1365-294X.2008.03712.x

Whitlock, M. C., and K. J. Gilbert, 2012   Qst in a hierarchically structured population. Mol. Ecol. Resour. 12: 481–483. https://doi.org/10.1111/j.1755-0998.2012.03122.x

Wickham, H., R. Francois, L. Henry, and K. Müller, 2017   dplyr: a grammar of data manipulation. R package version 0.7.4. Available from: https://cran.r-project.org/web/packages/dplyr/index.html

Yu, J., G. Pressoir, W. H. Briggs, I. Vroh Bi, M. Yamasaki *et al.*, 2006   A unified mixed-model method for association mapping that accounts for multiple levels of relatedness. Nat. Genet. 38: 203–208. https://doi.org/10.1038/ng1702

Zhao, H., Z. Sun, J. Wang, H. Huang, J.-P. Kocher *et al.*, 2013   Crossmap: a versatile tool for coordinate conversion between genome assemblies. Bioinformatics 30: 1006–1007. https://doi.org/10.1093/bioinformatics/btt730

Zhou, X., and M. Stephens, 2012   Genome-wide efficient mixed-model analysis for association studies. Nat. Genet. 44: 821–824. https://doi.org/10.1038/ng.2310

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Appendix

Appendix A—Environmental Variation and Inferences of Selection from Q_{PC}

Equation 1 assumes that there is no environmental variation contributing to fitness. It can be rewritten to include the effects of environmental variation as follows:

$$\overrightarrow{Z} \sim MVN(\mu, 2V_A K + V_E I), \tag{19}$$

where $\overrightarrow{Z}$ is a vector of traits with mean μ, I is the identity matrix, K is a kinship matrix, and V_E is a constant that measures environmental variation (Falconer and Mackay 1996; Hill 2010). Increases in V_E will thus increase the diagonal entries of the variance-covariance matrix for the multivariate normal distribution of $\overrightarrow{Z}$. The intuition behind the effect of V_E on the $\text{var}(\overrightarrow{Z})$ is that, in a properly designed common garden experiment, V_E will increase individual deviations from the expected trait value by increasing the diagonals of the variance-covariance matrix, but will not affect covariance between individuals.

Now, when we mean center $\overrightarrow{Z}$ and project it onto a matrix of the eigenvectors of the kinship matrix (U), we can get an expression for the set of projections ($\overrightarrow{z_m}$) across all eigenvectors:

$$\overrightarrow{z_m} = (\overrightarrow{Z} - \mu)U = U^T(\overrightarrow{Z} - \mu). \tag{20}$$

We can express the variance and distribution of X as

$$\text{Var}(\overrightarrow{z_m}) = U^T(2V_A K + V_E I)U, \tag{21}$$

$$\overrightarrow{z_m} \sim MVN(0, 2V_A \Lambda + V_E). \tag{22}$$

We can standardize z_m by the variance explained by each PC (the eigenvalues of K):

$$\overrightarrow{C} = \frac{\overrightarrow{z_m}}{\sqrt{\Lambda}} \tag{23}$$

$$\text{Var}(\overrightarrow{C}) = \Lambda^{-1/2}(2V_A \Lambda + V_E)(\Lambda^{-1/2})^T \tag{24}$$

$$\overrightarrow{C} \sim MVN\left(0, 1 + \frac{V_E}{2V_A}\Lambda^{-1}\right) \tag{25}$$

This result suggests that the contribution of V_E will be strongest along PCs with smaller eigenvalues ("later PCs"), so Q_{PC} is conservative in the face of V_E since it looks for an excess of differentiation along early PCs with larger eigenvalues compared to PCs with smaller eigenvalues.

We tested the intuition described above with simulations of traits that evolve neutrally for with $V_A = 1$ and $V_E = 0, 0.1$, and 0.5. We found that increasing V_E increased the variance of C_M at later PCs more than at early PCs (Figure S1A), and that this meant that fewer simulations showed significant signals of selection than would be expected under neutrality (Figure S1B)

Appendix B—Additive-by-Additive Epistasis and Q_{PC}

We denote the variance contributed by additive-by-additive epistasis as V_{AA}. Assuming no LD, we can rewrite Equation 1 as follows:

$$\overrightarrow{Z} \sim MVN(\mu, 2V_A K + 2V_{AA}K^2), \tag{26}$$

following *e.g.* Equation 9.13 in Falconer and Mackay (1996) and Hill (2010). Using the eigendecomposition of K, $K = U\Lambda U^{-1}$, where U is a matrix whose columns are the eigenvectors of K and Λ is a diagonal matrix with the eigenvalues of K, we find that $K^2 = U\Lambda^2 U^{-1}$. As in Appendix A, we can calculate the $\text{Var}(\overrightarrow{C})$ where $\overrightarrow{C}$ is a vector of the projections of $\overrightarrow{Z}$ onto U, standardized by dividing by $\Lambda^{-1/2}$.

$$\overrightarrow{C} \sim MVN\left(0, 1 + \frac{V_{AA}}{V_A}\Lambda\right) \tag{27}$$

Intuitively, we can see that, when V_{AA} is much larger than V_A, additive-by-additive epistasis will contribute disproportionately to variation along PCs that correspond to higher eigenvalues. Therefore, additive-by-additive epistasis that exceeds V_A can contribute to false positive signals of diversifying selection by increasing trait divergence along earlier PCs. However, in most situations, V_{AA} is unlikely to be large enough to significantly impact trait variance (Falconer and Mackay 1996; Hill 2010)

Appendix C—Mean Centering

Properly mean-centering conditional expectations for polygenic scores and the kinship matrix used to calculate Q_{PC} on these scores is crucial. However, the choice of how to properly mean-center these two parameters is not entirely straightforward when working with conditional distributions (as in Equation 10).

To illustrate the problem, imagine that we mean center the conditional expectations for polygenic scores in the genotyping panel ($\vec{\mu}'$) around the mean of the GWAS panel, such that $\vec{\mu}' = \mu_2 + K_{12}K_{22}^{-1}(\vec{X}_2 - \mu_2)$, where μ_2 is the mean polygenic score of individuals in the GWAS panel, $\vec{X}_2$ is the vector of polygenic scores in the GWAS panel, and K_{12} and K_{22} are subsets of the relatedness matrix between individuals in the genotyping panel and GWAS panel as defined for Equation 10. At the same time, we generate K_{11}, K_{22}, and K_{12} from the kinship matrix K following Equation 16, where K is mean centered around the combined mean of the genotyping and GWAS panel. While these two choices, made separately, seem intuitive, together they lead to a situation where, if $\mu_2 \neq \mu$, we can infer signals of adaptive divergence even if none exist. Therefore, we choose to mean center both K and $\vec{\mu}'$ around the mean of all individuals in the genotyping and GWAS panels.